STABILITY OF A JET OF VISCOELASTIC LIQUID IN THE PRESENCE OF A MASS FLUX AT ITS SURFACE

A. L. Yarin

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The stability of a jet of viscoelastic liquid with allowance for mass transfer at its surface is investigated by the method of small perturbations.

The action of surface-tension forces leads to the instability and breakup of jets of a dropping liquid having a small velocity relative to the surrounding gas. Rayleigh [1] investigated jets of an ideal liquid. The influence of the viscosity and shear elasticity of a liquid on the stability of a jet was studied in [2-8]. Recently installations have begun to be created which use jets of a dropping liquid at whose surface intense mass transfer takes place (see, e.g., [9]). However, detailed data are absent on the influence of one of the factors – the mass flux at the surface – on the breakup of a jet.

Estimation of the role of mass transfer is necessary when using jets of volatile substances and jets with chemical reactions and intense evaporation at the surface. The influence of mass transfer on the stability of a jet of dropping liquid is investigated theoretically below.

1. Let us consider a cylindrical laminar jet of incompressible Maxwellian liquid. The unperturbed cross section of the jet is a circle. At the surface of the jet there is a mass flux which does not depend on the curvature of the surface. The space outside the jet is filled with a nonviscous gas being liberated at the surface of the jet. We neglect the compressibility of the gas. We will also neglect the decrease in the radius of the jet, assuming that there is a mass source or sink at the surface of the jet. The value of the coefficient of surface tension of the liquid is assumed to correspond to the temperature at which the process of mass transfer under consideration takes place.

We use a cylindrical coordinate system moving with the jet whose axis coincides with the axis of the jet. The system of equations of continuity and motion

$$\operatorname{div}\mathbf{V}=0,\ \rho\frac{d\mathbf{V}}{dt}=\operatorname{Div}P \tag{1.1}$$

is closed inside the jet by the rheological equation of a Maxwellian liquid [8, 10],

$$\dot{S}=\frac{1}{2\eta}(P+p\delta)+\frac{1}{2\mu}\frac{\partial}{\partial t}(P+p\delta), \tag{1.2}$$

while outside the jet $P = -p\delta$; δ is a unit tensor.

The unperturbed steady-state solutions of the system (1.1) will be

$$v_{r1}=v_{\theta1}=v_{z1}=0,\ p_{r\theta1}=p_{\theta z1}=p_{rz1}=0, \tag{1.3}$$
$$p_1=p_{01},\ p_{rr1}=p_{\theta\theta1}=p_{zz1}=-p_{01}$$

inside the jet and

$$v_{r2}=\frac{ja}{\rho_2 r},\ v_{\theta2}=v_{z2}=0,\ p_2=-\frac{1}{2}\frac{j^2a^2}{\rho_2 r^2}+p_{02}+\frac{1}{2}\frac{j^2}{\rho_2} \tag{1.4}$$

outside it.

The connection between the pressures p_{01} and p_{02} is determined by the equation

$$p_{02}=p_{01}-\frac{\alpha}{a}-\frac{j^2}{\rho_2}. \tag{1.5}$$

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For small perturbations of the solutions (1.3) we obtain the following linearized system of equations from (1.1):

$$
\begin{gathered}
\frac{\partial v_r'}{\partial r} + \frac{1}{r}\frac{\partial v_\theta'}{\partial \theta} + \frac{\partial v_z'}{\partial z} + \frac{v_r'}{r} = 0,\\
\rho_1 \frac{\partial r v_r'}{\partial t} = \frac{\partial}{\partial r}(r p_{rr}') + \frac{\partial p_{r\theta}'}{\partial \theta} - p_{\theta\theta}' + \frac{\partial r p_{rz}'}{\partial z},\\
\rho_1 \frac{\partial v_\theta'}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r p_{r\theta}') + \frac{p_{r\theta}'}{r} + \frac{1}{r}\frac{\partial p_{\theta\theta}'}{\partial \theta} + \frac{\partial p_{\theta z}'}{\partial z},\\
\rho_1 \frac{\partial r v_z'}{\partial t} = \frac{\partial}{\partial r}(r p_{rz}') + \frac{\partial p_{\theta z}'}{\partial \theta} + \frac{\partial r p_{zz}'}{\partial z},\\
p_{rr}' = -p' + 2\eta \frac{\partial v_r'}{\partial r} - \frac{\eta}{\mu}\frac{\partial}{\partial t}(p_{rr}' + p'),\\
p_{\theta\theta}' = -p' + 2\eta\left(\frac{1}{r}\frac{\partial v_\theta'}{\partial \theta} + \frac{v_r'}{r}\right) - \frac{\eta}{\mu}\frac{\partial}{\partial t}(p_{\theta\theta}' + p'),\\
p_{zz}' = -p' + 2\eta \frac{\partial v_z'}{\partial z} - \frac{\eta}{\mu}\frac{\partial}{\partial t}(p_{zz}' + p'),\\
p_{rz}' = \eta\left(\frac{\partial v_z'}{\partial r} + \frac{\partial v_r'}{\partial z}\right) - \frac{\eta}{\mu}\frac{\partial p_{rz}'}{\partial t},\\
p_{r\theta}' = \eta\left(\frac{1}{r}\frac{\partial v_r'}{\partial \theta} + \frac{\partial v_\theta'}{\partial r} - \frac{v_\theta'}{r}\right) - \frac{\eta}{\mu}\frac{\partial p_{r\theta}'}{\partial t},\\
p_{\theta z}' = \eta\left(\frac{\partial v_\theta'}{\partial z} + \frac{1}{r}\frac{\partial v_z'}{\partial \theta}\right) - \frac{\eta}{\mu}\frac{\partial p_{\theta z}'}{\partial t}.
\end{gathered}
\tag{1.6}
$$

For perturbations which depend exponentially on time the system of equations (1.6) is reduced by the method of Landau [11] to the equation

$$\Delta p' = 0. \tag{1.7}$$

A solution of (1.7) finite at r = 0 will be

$$p' = A I_s(kr)\exp(ikz + is\theta + i\sigma t). \tag{1.8}$$

The expressions for the perturbations of the remaining quantities are found by solving a system of 10 ordinary differential equations obtained from (1.6) [all the perturbations contain the factor $\exp(ikz + is\theta + i\sigma t)$]. We have

$$
\begin{gathered}
v_r' = \frac{1}{2}\{B_1 I_{s+1}(lr) + B_2 I_{s-1}(lr) + C[I_{s+1}(kr) + I_{s-1}(kr)]\}\exp(ikz + is\theta + i\sigma t),\\
v_\theta' = \frac{1}{2i}\{B_1 I_{s+1}(lr) - B_2 I_{s-1}(lr) + C[I_{s+1}(kr) - I_{s-1}(kr)]\}\exp(ikz + is\theta + i\sigma t),\\
v_z' = \Big[\frac{i}{2k}\Big\{B_1 l I_{s+1}'(lr) + B_2 l I_{s-1}'(lr) + Ck[I_{s+1}'(kr) + I_{s-1}'(kr)] + \frac{B_1 I_{s+1}(lr) + B_2 I_{s-1}(lr) + C[I_{s+1}(kr) + I_{s-1}(kr)]}{r}\Big\}\\
+ \frac{si}{2kr}\{B_1 I_{s+1}(lr) - B_2 I_{s-1}(lr) + C[I_{s+1}(kr) - I_{s-1}(kr)]\}\Big]\exp(ikz + is\theta + i\sigma t)
\end{gathered}
\tag{1.9}
$$

[the expressions for the perturbations of the stresses are obtained by substituting Eqs. (1.9) into the last six equations of (1.6)]. Here

$$l^2 = k^2 + \rho_1 i\sigma\left(\frac{1}{\eta} + \frac{i\sigma}{\mu}\right), \quad C = -\frac{Ak}{\rho_1 i\sigma}. \tag{1.10}$$

In (1.9) and later a prime denotes the derivatives with respect to the entire argument of the modified Bessel functions.

The small perturbations of the solutions (1.4) must be determined from the linearized equations of continuity [the first equation in (1.6)] and motion:

$$\rho_2\left(\frac{\partial v_r'}{\partial t}+\frac{C_1}{r}\frac{\partial v_r'}{\partial r}-\frac{C_1}{r^2}v_r'\right)=-\frac{\partial p'}{\partial r},$$
$$\rho_2\left(\frac{\partial v_\theta'}{\partial t}+\frac{C_1}{r}\frac{\partial v_\theta'}{\partial r}+\frac{C_1}{r^2}v_\theta'\right)=-\frac{1}{r}\frac{\partial p'}{\partial \theta}, \tag{1.11}$$
$$\rho_2\left(\frac{\partial v_z'}{\partial t}+\frac{C_1}{r}\frac{\partial v_z'}{\partial r}\right)=-\frac{\partial p'}{\partial z}.$$

Here

$$C_1=Ua. \tag{1.12}$$

The method of Landau [11] cannot be used to determine the perturbations in the region filled with gas, since there are coefficients to the partial derivatives which depend on r on the left sides of (1.11).

In this case the pressure can be eliminated from (1.11), after which there is no difficulty in determining the perturbations in the region filled with gas. Details are given in the Appendix. The results have the form

$$v_r'=\left\{\frac{HK_s'}{i}+D\left[\frac{2}{ik^2}(V_1I_s'-V_4K_s')-\frac{\sigma}{C_1k^4}(V_2I_s'-V_5K_s')-\frac{s^2C_1}{\sigma}(V_3I_s'-V_6K_s')-\frac{r}{ik}\exp\left(-\frac{i\sigma r^2}{2C_1}\right)\right]\right.$$
$$\left.+E\frac{sC_1k}{\sigma}(V_3I_s'-V_6K_s')\right\}\exp(ikz+is\theta+i\sigma t),$$
$$v_\theta'=\left\{\frac{s}{kr}HK_s+D\frac{s}{kr}\left[\frac{2}{k^2}(V_1I_s-V_4K_s)-\frac{i\sigma}{C_1k^4}(V_2I_s-V_5K_s)\right.\right.$$
$$\left.+\frac{s^2C_1}{i\sigma}(V_3I_s-V_6K_s)+\frac{C_1}{i\sigma}\exp\left(-\frac{i\sigma r^2}{2C_1}\right)\right] \tag{1.13}$$
$$\left.-E\left[\frac{s^2C_1}{ri\sigma}(V_3I_s-V_6K_s)+\frac{C_1}{ri\sigma}\exp\left(-\frac{i\sigma r^2}{2C_1}\right)\right]\right\}\exp(ikz+is\theta+i\sigma t),$$
$$v_z'=\left\{HK_s+D\left[\frac{2}{k^2}(V_1I_s-V_4K_s)-\frac{i\sigma}{C_1k^4}(V_2I_s-V_5K_s)\right.\right.$$
$$\left.\left.+\frac{s^2C_1}{i\sigma}(V_3I_s-V_6K_s)\right]-E\frac{sC_1k}{i\sigma}(V_3I_s-V_6K_s)\right\}\exp(ikz+is\theta+i\sigma t).$$

Here the modified Bessel functions I_s and K_s have the argument $r_1=kr$ and

$$V_1(r_1)=\int_\infty^{r_1}r_1K_s(r_1)\exp(-Wr_1^2)\,dr_1,\quad V_2(r_1)=\int_\infty^{r_1}r_1^3K_s(r_1)\exp(-Wr_1^2)\,dr_1,$$
$$V_3(r_1)=\int_\infty^{r_1}\frac{1}{r_1}K_s(r_1)\exp(-Wr_1^2)\,dr_1,\quad V_4(r_1)=\int_\infty^{r_1}r_1I_s(r_1)\exp(-Wr_1^2)\,dr_1, \tag{1.14}$$
$$V_5(r_1)=\int_\infty^{r_1}r_1^3I_s(r_1)\exp(-Wr_1^2)\,dr_1,\quad V_6(r_1)=\int_\infty^{r_1}\frac{1}{r_1}I_s(r_1)\exp(-Wr_1^2)\,dr_1,\quad W=i\sigma/2\,C_1k^2.$$

The perturbation of the pressure is determined from any projection of the equation of motion (1.11) after the substitution of Eqs. (1.13).

2. The conditions of constancy and continuity of the mass flux at the surface of the jet have the form

$$v_{r1}'=\frac{\partial\zeta}{\partial t},\quad v_{r1}'=v_{r2}'-\frac{\zeta}{a}\frac{j}{\rho_2}\quad \text{at}\quad r=a. \tag{2.1}$$

(In these and subsequent conditions at the surface of the jet small quantities of higher orders are omitted.) Because of the continuity of the r component of the momentum at the surface of the jet,

$$-p_{rr1}'-p_2'=-\frac{\alpha}{a^2}\left(\zeta+\frac{\partial^2\zeta}{\partial\theta^2}\right)-\alpha\frac{\partial^2\zeta}{\partial z^2}+\frac{\zeta j^2}{a\rho_2}\quad \text{at } r=a. \tag{2.2}$$

The continuity of the tangential velocity component at the surface of the jet is expressed by the equations

$$v_{\theta1}'=v_{\theta2}'+\frac{j}{\rho_2a}\frac{\partial\zeta}{\partial\theta},\quad v_{z1}'=v_{z2}'+\frac{j}{\rho_2}\frac{\partial\zeta}{\partial z}\quad \text{at}\quad r=a, \tag{2.3}$$

while the conditions of continuity of the θ and z components of the momentum at the surface of the jet have the form

$$p'_{r\theta 1} = 0, \ p'_{rz1} = 0 \quad \text{at} \ r = a. \tag{2.4}$$

Substituting the expressions for the perturbations inside and outside the jet into the conditions (2.1)-(2.4), we obtain the dispersion equation of the problem.

3. Let us consider axisymmetric perturbations of the jet ($s = 0$, $B_1 = B_2$, $E = 0$) and find the solutions of the dispersion equation in the long-wave approximation. In this case $W \gg 1$.

For a jet with a small Ohnesorge number ($\sqrt{M} \ll 1$) the solution of the dispersion equation is sought in the form of a series $d = d_0 + \sqrt{M}d_1 + \dots$. Taking $Z = \gamma/\sqrt{M}$ and discarding small quantities of higher orders, of the order $\sqrt{M}n$, in particular, we find

$$d_0 = -\frac{1}{2}\frac{\rho_2}{\rho_1} n \frac{I_1}{K_1} \frac{(K_0 - \chi K_1)}{[I_0 + \rho_2 K_0 I_1/(\rho_1 K_1)]} \pm \sqrt{\frac{\chi(1-\chi^2) I_1}{I_0 + \rho_2 K_0 I_1/(\rho_1 K_1)}}, \tag{3.1}$$

$$d_1 = -\frac{I_0(1-\chi^2) I_1 \chi^3/d_0 + d_0 \chi^2 I_1' [I_0 + \rho_2 K_0 I_1/(\rho_1 K_1)]}{d_0(1+d_0\gamma)[I_0 + \rho_2 K_0 I_1/(\rho_1 K_1)]^2},$$

where $n = \sqrt{\rho_1 a/\alpha}U$.

Here and later all the modified Bessel functions have the argument $\chi = ka$. According to the Rayleigh hypothesis, a jet breaks up under the action of the perturbation which corresponds to the maximum positive value of d. In the given case the fastest growing perturbation corresponds to a value of $\chi_0 \approx 1/\sqrt{2} \approx 0.7$ [the plus sign in the first equality of (3.1)]. The release of gas at the surface of the jet ($n > 0$) leads to an increase in the maximum positive value of d and hence to destabilization, since $K_0 - \chi K_1 < 0$ when $\chi > 0.6$. Conversely, the absorption of gas at the surface of the jet ($n < 0$) shows the growth of the fastest growing perturbations. Consequently, the release of gas leads to a decrease and the absorption of gas to an increase in the length of the unbroken-up jet. Here and later in comparing the lengths of jets it is assumed that the coefficients of surface tension of the liquids are equal and the discharge conditions are the same.

The value of $\chi_0 = 1/\sqrt{2}$ corresponds to the length of the fastest growing perturbation in the absence of mass transfer at the surface of the jet. An investigation of the dependence $d_0 = d_0(\chi)$ obtained shows that $\chi_0 > 1/\sqrt{2}$ when $n > 0$ while $\chi_0 < 1/\sqrt{2}$ when $n < 0$. Therefore, the release of gas at the surface of the jet leads to a decrease in the size of the drops into which the jet breaks up, while gas absorption leads to an increase in it.

Positive values of d_0 correspond to negative values of d_0 when $\chi < 1$, with the absolute value of d_1 being less for a Maxwellian liquid ($\gamma > 0$) than for a Newtonian liquid ($\gamma = 0$). Consequently, elastic effects in a viscous liquid lead to a decrease in the length of the jet before its breakup. Then a jet of viscoelastic liquid is longer than a jet of an ideal liquid. A similar result was obtained from a linear analysis of the breakup in [6] without allowance for mass transfer at the surface of a jet. Nonlinear effects can lead to considerable retardation of the breakup of a jet of viscoelastic liquid and to an increase in its length [6, 7].

We note that the breakup of a jet occurs just because of the growth of axisymmetric oscillations. In fact, in the general case of asymmetric perturbations of a jet relative to the axis the dispersion equation of the problem is reduced to

$$d^2 = \frac{\chi(1-s^2-\chi^2)(I_{s+1}+I_{s-1})}{2\{I_s + \rho_2 K_s (I_{s+1}+I_{s-1})/[2\rho_1(K_{s-1}+sK_s/\chi)]\}} + O\left[M^{q_1}\left(n\frac{\rho_2}{\rho_1}\right)^{q_2}\psi(\chi, d, s)\right], \tag{3.2}$$

where $M^{q_1}\left(n\frac{\rho_2}{\rho_1}\right)^{q_2} \ll 1$; $\psi(\chi, d, s) = O(1)$. When $s \geq 1$ the real part of d cannot be larger in order of magnitide than $M^{q_1}(n\rho_2/\rho_1)^{q_2}$, and hence the growth of these perturbations is slower than the growth of axisymmetric perturbations.

For a jet with an Ohnesorge number $\sqrt{M} \gg 1$ the dispersion equation of the problem has the solution

$$f = \frac{1-\chi^2}{6 - Z + [K_0/(\chi K_1) - 1]\rho_2 \mathrm{Re}/\rho_1} \tag{3.3}$$

[here Z is no larger than one in order of magnitude, and the small quantities $Z^{q_1}\left(\frac{\rho_2}{\rho_1}\mathrm{Re}\right)^{q_2}\left(\frac{1}{M}\right)^{q_3}$ are omitted]. If the liquid is Newtonian ($Z = 0$) and there is no mass transfer at the surface of the jet ($\mathrm{Re} = 0$), then Eq. (3.3) coincides with the result obtained by Veber [2]. The presence of elastic effects in the liquid leads to an increase in the rate of growth of perturbations and hence to shortening of the jet (if nonlinear effects are unimportant). The expression for the length of a jet before breakup obtained using (3.3) with Re = 0 coincides with the result of [8], in which the stability of a jet of viscoelastic liquid with the rheological equation (1.2) without allowance for mass transfer is investigated. The release of gas at the surface of a jet destabilizes perturbations for which $\chi > 0.6$ and stabilizes them when $\chi < 0.6$. Gas absorption leads to the opposite effect. A similar result was obtained for a jet with a small Ohnesorge number (3.1). Thus, gas release at the surface of a jet stabilizes it with respect to perturbations with a very long wavelength and destabilizes it with respect to other perturbations.

The explanation for this is that the generation of vortices in the surrounding gas which destabilize the jet is very small in the case of perturbations with a very long wavelength, and hence the stabilizing effect of the velocity field of the main flow outside the jet predominates. With a decrease in the wavelength of the disturbance the generation of vortices increases, which leads to the growth of perturbations in the final analysis. The opposite effects are observed with gas absorption at the surface of the jet.

The influence of the indicated processes on the breakup of a jet depends on what wavelength corresponds to the fastest growing perturbation. Therefore, when $\sqrt{M} \ll 1$ gas release accelerates and gas absorption retards the breakup of a jet, in contrast to the case of $\sqrt{M} \gg 1$, where gas release and absorption have the opposite effects.

We note that the direct influence of mass transfer on the stability of a jet is small, since it is determined by terms of order $\rho_2/\rho_1 \ll 1$ in (3.1)-(3.3), whereas the contribution of terms connected with other effects is of order unity. Therefore, only a change in the coefficient of surface tension, taking place in processes connected with mass transfer, as a rule, can appreciably affect the length of the unbroken-up part of the jet, since $L \sim (d\sqrt{\alpha})^{-1}$ when $\sqrt{M} \ll 1$ and $L \sim (f\alpha)^{-1}$ when $\sqrt{M} \gg 1$, while the coefficient of surface tension is taken at the temperature characteristic of the process of mass transfer under consideration.

In the case of a vanishingly low surface tension of the liquid (an ideal liquid, for simplicity) the characteristic time $\sqrt{\rho_1 a^3/\alpha}$ of capillary breakup can prove to be greater than the characteristic time $[\rho_2 Uk/\rho_1]^{-1}$ of breakup of the jet due to mass transfer. In this case the dispersion equation of the problem has the solution

$$w = \frac{I_1[1-K_0/(\chi K_1)]\,\mathrm{sign}\,U}{2[I_0+\rho_2 K_0 I_1/(\rho_1 K_1)]} \pm \sqrt{\left\{\frac{I_1[1-K_0/(\chi K_1)]}{2[I_0+\rho_2 K_0 I_1/(\rho_1 K_1)]}\right\}^2 + \frac{T(1-\chi^2)I_1}{\chi[I_0+\rho_2 K_0 I_1/(\rho_1 K_1)]}}, \tag{3.4}$$

where $T = \alpha\rho_1/\rho_2^2 U^2 a$.

The influence of mass transfer on the stability of a jet is analogous to that obtained in the solutions (3.1) and (3.3). In particular, if

$$T\,\frac{(1-\chi^2)I_1}{\chi[I_0+\rho_2 K_0 I_1/(\rho_1 K_1)]} \ll \left\{\frac{I_1[1-K_0/(\chi K_1)]}{2[I_0+\rho_2 K_0 I_1/(\rho_1 K_1)]}\right\}^2, \tag{3.5}$$

then

$$w = \frac{I_1[1-K_0/(\chi K_1)]\,\mathrm{sign}\,U}{I_0+\rho_2 K_0 I_1/(\rho_1 K_1)}. \tag{3.6}$$

A jet for which the condition (3.5) is satisfied proves to be stable with respect to perturbations with $\chi < 0.6$ if gas is released at its surface and unstable if gas is absorbed. When $\chi > 0.6$ the gas release destabilizes the jet while absorption stabilizes it.

APPENDIX

4. Differentiating the first of equations (1.11) with respect to z and the third with respect to r and subtracting the resulting equations, we find the equation for θ, the projection of the curl of a velocity perturbation:

$$\frac{\partial\Omega_1}{\partial t} + \frac{C_1}{r}\frac{\partial\Omega_1}{\partial r} = \frac{C_1}{r^2}\Omega_1,\quad \Omega_1 = \frac{\partial v_z'}{\partial r} - \frac{\partial v_r'}{\partial z} = -\mathrm{curl}_\theta \mathbf{V}'. \tag{4.1}$$

Similarly, from system (1.11) we obtain

$$\begin{gathered}\frac{\partial\Omega_2}{\partial t} + \frac{C_1}{r}\frac{\partial\Omega_2}{\partial r} = \frac{C_1}{r^2}\Omega_2,\quad \Omega_2 = \frac{\partial v_\theta' r}{\partial r} - \frac{\partial v_r'}{\partial\theta} = r\,\mathrm{curl}_z \mathbf{V}',\\ \frac{\partial\Omega_3}{\partial t} + \frac{C_1}{r}\frac{\partial\Omega_3}{\partial r} = -\frac{C_1}{r^2}\Omega_3,\quad \Omega_3 = \frac{1}{r}\frac{\partial v_z'}{\partial\theta} - \frac{\partial v_\theta'}{\partial z} = \mathrm{curl}_r \mathbf{V}'.\end{gathered} \tag{4.2}$$

The first-order, linear, inhomogeneous equations (4.1) and (4.2) describe the conservation of curl of a velocity perturbation in an ideal liquid and can be obtained directly by linearizing the Helmholtz equation.

The solutions of the Cauchy problem for Eqs. (4.1) and (4.2) with the initial conditions

$$\Omega_m = \varphi_m(r), \ m = 1, 2, 3 \ \text{at} \ t = 0 \tag{4.3}$$

have the form

$$\Omega_1 = \frac{r}{\beta}\varphi_1(\beta), \ \Omega_2 = \frac{r}{\beta}\varphi_2(\beta), \ \Omega_3 = \frac{\beta}{r}\varphi_3(\beta), \tag{4.4}$$

where $\beta = \sqrt{2(r^2/2 - C_1 t)}$. The functions φ_m contain θ and z as parameters.

The expressions (4.4) allow one to determine the velocity perturbation field for any initial perturbation. The solution of the Cauchy problem can be constructed similarly in an investigation of the hydrodynamic stability of liquid combustion (a burning half plane).

In the given case one must be confined to consideration of three functions φ_m which give the same dependence on t, θ, and z of perturbations in the region filled with gas as was already obtained inside the jet [Eqs. (1.9)]. We have

$$(\Omega_1, \Omega_2, \Omega_3) = \left(Dr, Er, \frac{Q}{r}\right) \exp\left(-\frac{i\sigma r^2}{2C_1} + ikz + is\theta + i\sigma t\right). \tag{4.5}$$

Representing the velocity perturbations in the form

$$(v_r', v_\theta', v_z') = (F_1(r), F_2(r), F_3(r)) \exp(ikz + is\theta + i\sigma t), \tag{4.6}$$

from (4.5) and the first equation (1.6), the continuity equation, we obtain a system of four ordinary differential equations with the unknown functions F_1, F_2, and F_3. Then the condition

$$Q = (Ek - sD)\frac{C_1}{\sigma} \tag{4.7}$$

must be satisfied, and the system obtained is reduced to the equation

$$F_3'' + \frac{F_3'}{r_1} - \left(1 + \frac{s^2}{r_1^2}\right)F_3 = \left[\frac{2D}{k^2} - D\frac{i\sigma}{C_1}\frac{r_1^2}{k^4} - \frac{sC_1k}{i\sigma}\left(E - \frac{s}{k}D\right)\frac{1}{r_1^2}\right]\exp\left(-\frac{i\sigma r_1^2}{2C_1k^2}\right), \tag{4.8}$$

where $r_1 = kr$ and $F_3^{(m)} = d^mF_3/dr_1^m$. The solution of (4.8) which is bounded as $r \to \infty$ is found by the Lagrange method. Thus, we obtain Eqs. (1.13) for perturbations in the region filled with gas.

NOTATION

r, θ, z, cylindrical coordinates; t, time; V, velocity vector with projections v_r, v_θ, v_z; P, stress tensor with components p_{rr}, $p_{\theta\theta}$, p_{zz}, $p_{r\theta}$, p_{rz}, $p_{\theta z}$; $\dot{S}$, tensor of strain velocities; p, pressure; ρ, density; η, coefficient of viscosity; μ, Lamé coefficient; α, coefficient of surface tension; U, radial velocity of gas at surface of unperturbed jet ($U = j/\rho_2$); j, mass flux at surface of jet; k, s, wave numbers of a perturbation; σ, frequency of perturbations; p_{0g} (g = 1, 2), pressure at surface of unperturbed jet; a, radius of unperturbed jet; L, length of jet before breakup; A, B_1, B_2, C, D, E, Q, H, constant coefficients; $Re = \rho_1 Ua/\eta$; $M = \eta^2/\rho_1\alpha a$; $Z = \alpha/\mu a$; $d = i\sigma/\sqrt{\alpha/\rho_1 a^3}$; $w = i\sigma/\rho_2|U|k/\rho_1$; $f = i\sigma/(\alpha/\eta a)$; $\gamma = 0$ corresponds to a Newtonian liquid; $\gamma > 0$ corresponds to a viscoelastic liquid. The indices 1 and 2 correspond to regions inside and outside the jet. Perturbations are marked by primes.

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INTENSIFICATION OF CONVECTIVE HEAT EXCHANGE BY RIBBON SWIRLERS IN THE FLOW OF ANOMALOUSLY VISCOUS LIQUIDS IN PIPES

Yu. G. Nazmeev

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The results are given on an experimental investigation of the intensification of convective heat exchange in anomalously viscous media through the use of inserts of twisted ribbon.

The intensification of convective heat exchange in pipes and channels of heat-exchange apparatus is a most important problem for many branches of industry. Solving this problem enables one to reduce the size of heat-exchange apparatus and to increase their output.

A well-known means of intensifying convective heat exchange in pipes is the use of helical inserts of twisted ribbon. By now extensive experimental material has been accumulated on heat exchange in pipes containing helical intensifiers [1-5]. All the available test data pertain to the case of the flow of viscous liquids, however. In this case it is seen from an analysis of the well-known reports [1, 5-7] that the use of ribbon swirlers intensifies heat exchange in viscous liquids by up to 2.5 times, with the largest increase in the coefficients of heat transfer being observed in the region of Reynolds numbers from 3000 to 6500. The cause of the increase in heat transfer in viscous liquids is the formation and development of secondary flows of the first and partly of the second kind under the action of centrifugal forces. In addition, the use of an insert of twisted ribbon increases the heat-exchange surface and an increase in heat transfer also occurs due to the ribbing effect. With an increase in the Reynolds number above 6500 turbulence has the prevailing effect on the intensity of heat transfer, while the role of secondary flows decreases.

Unfortunately, up to now test data are entirely absent on the intensification of heat exchange by the indicated means in the flow of anomalously viscous liquids, which find very wide application in modern technology.

On the basis of the fact that the use of ribbon swirlers provides a considerable gain in heat transfer in the flow of viscous liquids, in the present report an attempt was made to experimentally determine the possibilities for the intensification of heat exchange in anomalously viscous media using the given swirlers, as well as to estimate the efficiency of their use.

The tests were conducted on the experimental installation described in [8]. A pipe of 1Kh18N10T stainless steel with an inner diameter of 12 mm and a length of 1200 mm was used as the working section. The treatment of the inner surface of the pipe corresponded to the eighth class of purity. The tests were conducted with helical inserts of twisted brass ribbon 0.5 mm thick. The pitch of the ribbon swirlers (in a rotation of the ribbon by 180°) lay in the range from 38 to 600 mm. All the tests were conducted under steady thermal and hydrodynamic conditions.

Tests with water and transformer oil, which showed good convergence with the well-known generalizing criterial equation of [6] were made preliminarily on the experimental installation. Aqueous solutions of sodium carboxymethylcellulose (Na-CMC) of different concentrations were used as the model anomalously viscous liquids. The rheological characteristics of the model liquids were determined on a Rheotest rotary viscosimeter and on a Höppler viscosimeter. The results of the viscosimetric measurements of two model liquids in the temperature range from 20 to 80°C are presented in Fig. 1. The thermophysical characteristics of the anomalously viscous liquids were determined in accordance with [9, 10]. The results of the thermophysical measurements are given in Table 1.

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